International Journal of Latest Trends in Engineering and Technology
Vol.(14)Issue(1), pp.058-062
DOI: http://dx.doi.org/10.21172/1.141.11
e-ISSN:2278-621X

A STUDY ON SOFT GENERALIZED REGULAR CLOSED SETS

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Abstract: Generalized closed sets are playing a vital role in the study of topology as well as fuzzy topology and there are several generalizations existing in the literature. The so called generalized regular closed set has been introduced by S. Bhattacharya [3] and J. Chakraborty et. al. [2] in a given topological space and fuzzy topological space respectively. This paper intends to introduce the concept of soft generalized regular closed (in short, gr-closed) sets and to establish some relations between the soft gr-closed sets and some other kinds of soft generalized sets such as soft regular closed sets, soft generalized closed sets and also with that of soft regular generalized closed sets. Finally, some applications are drawn via respective soft generalized continuities.
Keywords : Soft gr-closed set, soft regular closed set, soft generalized closed set, soft gr-continuous, soft gr-irresolute.

1. INTRODUCTION

In 1999, Molodtsov [9] introduced the concept of soft set theory in order to deal with the uncertainties in complicated real life problems. He successfully applied this theory in various directions such as game theory, operation research, probability, smoothness of functions etc., while many other theories have been studied before to deal with these uncertainties, such as theory of probability, theory of vague sets, fuzzy set theory, theory of rough sets, which are considered as mathematical tools to rectify those uncertainties but the theories have their own limitations. Thereafter, applications of soft set theory took a pace in various other fields.

In 2011, Shabir and Naz [11] introduced the notion of soft topological spaces, which is defined over an initial universe with a fixed set of parameters which generates a parameterized family of topological spaces and introduced some of the basic notions such as soft open sets, soft closed sets, soft closure, soft interior , soft subspace, soft Ti- spaces, i = 1; 2; 3; 4, soft regular spaces, soft normal spaces. In the same year, many other researchers [4], [8], etc., continued the work and strengthened the theory of soft topological spaces. In 1993, N. Palaniappan [10] found an interesting theory of regular generalized closed sets in a topological space. Later on in 2014, Yuksel et. al. [12] defined the notion of soft regular generalized closed sets in a soft topological space. In this paper, we study the concept of soft generalized regular closed sets which was initially proposed by S. Bhattacharya [3] in ordinary topological spaces followed by J, Chakraborty et. al.[2] in fuzzy topological spaces in the year 2015. Also, we study the relationships of this newly defined set with some of the existing sets in the same space.

2. PRELIMINARIES

N. Levine [6] first introduced the idea of generalized closed sets in a topological space in 1970 and since its inception the idea was extended in various environments for different objectives. Before going to the main results, we need some preliminary ideas those are essentially required for the study.

Definition 2.1. [9] Let X be an initial universe and E be a set of parameters. Let P (X) denote the power set of X and A be a non-empty subset of E. A pair FA is called a soft set over X, where F is a mapping given by F : A ! P (X).

In other words a soft set over X is a parameterized family of subsets of the universe X. For e 2 A, F (e) may be considered as the set of e-approximate elements of the soft set FA.

Definition 2.2. [11] Let X be an initial universe set and E be the fixed non-empty set of parameters with respect to X unless otherwise specified.

Let be the collection of soft sets over X, thenis said to be a soft topology on X if

1) e, Xe 2 .
2) The union of any number of soft sets in belongs to .
3) The intersection of any two soft sets in belongs to .

Thus, the triplet (X; ; E) is called a soft topological space over X. The members of are called soft open sets in X and if their relative complements also belong to , then they are termed as soft closed sets. We denote the collection of all soft open sets in X by O(X; ; E) and collection of all soft closed sets by C(X; ; E).

Definition 2.3. [11] Let (X; ; E) be a soft topological space over X and FE be a soft set over X. Then the soft closure of FE, denoted by clFE is the intersection of all soft closed sets containing FE. Clearly, clFE is the smallest soft closed set over X which contains FE.

Theorem 2.4. [11] Let (X; ; E) be a soft topological space and let FE and GE be the soft sets over X. Then

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1) cl e = e and clXe = Xe.
2) FE e clFE:
3) FE is a soft closed set iff FE = clFE:
4) cl(clFE) = clFE:
5) FE e GE implies clFE e clGE.

Definition 2.5. [11] Let (X; ; E) be a soft topological space over X and FE be a soft set over X. Then the soft interior of FE, denoted by intFE, is the union of all soft open sets contained in FE. Clearly, intFE is the largest soft open set over X contained in FE.

Theorem 2.6. [4] Let (X; ; E) be a soft topological space over X and FE and GE are soft sets over X. Then

1) inte = e and intXe = Xe.
2) intFE e FE:
3) FE is a soft open set iff intFE = FE:
4) int(intFE) = intFE:
5) FE e GE implies intFE e intGE.

Definition 2.7. [5] Let (X; ; E) be a soft topological space. Then a soft subset FE of X is called soft generalized closed or, briefly soft g-closed , if clFE e GE, whenever FE e GE, where GE is a soft open set in X. The complement of a soft g-closed set is a soft g-open set.

Definition 2.8. [12] A soft subset FE of X is said to be soft regular open if FE = int(clFE) and soft regular closed if FE = cl(intFE).

Definition 2.9. [12] Let (X; ; E) be a soft topological space. A soft set FE is called soft regular generalized closed (briefly, soft rg-closed) in X if and only if clFE e GE, whenever FE e GE and GE is soft regular open in X.

Theorem 2.10. [12] Let (X; ; E) be a soft topological space and FE a soft set over X. If FE is soft generalized closed, then FE is a soft rg-closed set.

3. SOFT GENERALIZED REGULAR CLOSED SETS

In this section, we introduce the notion of soft generalized regular closed set and characterize some basic and fundamental properties of it. Moreover, we establish some interrelationships among Soft gen-eralized regular closed sets, soft regular closed sets, soft generalized closed sets, soft closed sets and soft regular generalized closed sets.

Definition 3.1. Let FE be a soft subset of a soft topological space (X; ; E), then
clr(FE) = fGE : GE e FE; GE is a soft regular closed set in Xg.

Definition 3.2. Let FE be a soft subset of a soft topological space (X; ; E), then
intr(FE) = fGE : GE e FE; GE is a soft regular open set in Xg.

Definition 3.3. A soft set FE in a soft topological space (X; ; E) is said to be a soft generalized regular closed (or in brief, soft gr-closed) set if clr(FE) e GE, whenever FE e GE and GE is soft open in X. Moreover, complement of a soft gr-closed set is said to be a soft gr-open set.

Example 3.4. Let X = fh1; h2; h3g; E = fe1; e2g and = fe; X;e FE1 ; FE2 g, where FE1 ; FE2 are soft sets on X defined by
F 1(e1) = fh1g; F 1(e2) = fh2g;
F 2(e1) = fh2; h3g; F 2(e2) = fh1; h3g
Let FE be a soft set over X defined by F (e1) = fh2g; F (e2) = fh1g, then FE is a soft gr-closed set, since for the soft open set FE2 in X, clrFE e FE2 as FE e FE2 .

Remark 3.5. Finite intersection and finite union of soft gr-closed sets are not soft gr-closed. The following Example 3.6 and Example 3.7 clears the remark respectively.

Example 3.6. Let X = fa; b; c; d; eg; E = fe1; e2g and = fe; X;e FE1 ; FE2 ; FE3 g, where FE1 ; FE2 and FE3 are soft sets on X defined by
F 1(e1) = fa; bg; F 1(e2) = fb; dg
F 2(e1) = fcg; F 2(e2) = feg
F 3(e1) = fa; b; cg; F 3(e2) = fb; d; eg.
Here, is a soft topology and (X; ; E) is a soft topological space. Let us consider two soft sets GE and HE on X respectively defined by
G(e1) = fa; c; dg; G(e2) = fa; b; eg and H(e1) = fb; c; eg; H(e2) = fa; d; eg are soft gr-closed sets of X.
But GE \ HE = SE, where SE is defined as S(e1) = fcg; S(e2) = feg, which is not a soft gr-closed set.
Thus, finite intersection of soft gr-closed set is not a soft gr-closed set.

Example 3.7. Consider Example 3.6. Let = fe; X;e FE1 ; FE2 ; FE3 ; FE4 g,where FE1 ; FE2 ; FE3 and FE4 are soft sets on X, defined by
F 1(e1) = fa; bg; F 1(e2) = fb; cg
F 2(e1) = fcg; F 2(e2) = fdg
F 3(e1) = fa; b; cg; F 3(e2) = fb; c; dg
F 4(e1) = fd; eg; F 4(e2) = fa; eg.

Then is a soft topology and (X; ; E) is a soft topological space. Let us suppose two soft sets GE and HE in X respectively defined by G(e1) = fdg; G(e2) = fag and H(e1) = feg; H(e2) = feg. One can easily prove that GE and HE are soft gr-closed sets but GE [HE = SE, where SE is defined as S(e1) = fd; eg; S(e2) = fa; eg is not a soft gr-closed set. Thus, union of two soft gr-closed set is not a soft gr-closed set.
Theorem 3.8. If a soft set in a soft topological space (X; ; E) is soft generalized regular closed set then it is a soft generalized closed set.
Proof. If FE is a soft gr-closed set in X, then clrFE GE, whenever FEGE and GE is soft open.
clFE clrFE clFE GE FE ise softclosed set.
Also, . It implies . Thus, generalized e □
e e

Remark 3.9. A soft generalized closed set in a soft topological space X may not be a soft gr-closed set, as it is verified in the following example.
Example 3.10. Let X = fa; b; cg; E = fe1; e2g and = fe; X;e FE1 ; FE2 ; FE3 g, where FE1 ; FE2 and FE3 are soft sets on X defined by
F 1(e1) = fa; bg; F 1(e2) = fa; cg
F 2(e1) = fb; cg; F 2(e2) = fb; cg
F 3(e1) = fbg; F 3(e2) = fcg.
Here, is a soft topology and (X; ; E) is a soft topological space. Now let FE be a soft set in X defined by
F (e1) = fcg; F (e2) = fbg. Clearly, FE is a soft g-closed set but not a soft gr-closed subset in X.
Theorem 3.11. If a soft set is a soft regular closed set in (X; ; E), then it is soft gr-closed set in (X; ; E).
Proof. The proof is straightforward and hence omitted. □
Remark 3.12. A soft gr-closed set in a soft topological space X may not be a soft regular closed. which can be observed by the follwoing example. The converse of the above theorem is not always true, it can be shown by the following example.
Example 3.13. Let X = fa; b; cg; E = fe1; e2 g and = fe; X;e FEg, where FE is a soft set on X defined by F (e1) = fa; bg; F (e2) = fb; cg: Here, is a soft topology and (X; ; E) is a soft topological space. Let GE be a soft set on X, defined as G(e1) = fa; cg; G(e2) = fa; bg. Here, GE is a soft gr-closed set but not a soft closed set.
Nonetheless, in Theorem 3.15, the implication is obtained with some additional condition.
Remark 3.14. e and Xe are soft gr-closed subset of X.
Theorem 3.15. If FE is a soft open and a soft gr-closed set in (X; ; E) then it is a soft regular closed set in X.
Proof. Let FE be a soft open set as well as a soft gr-closed set in X. Then intFE = FE, as FE is a soft open set and also by the definition of soft gr-closed set, clrFE intFE = FE. But we know that
FE e clrFE, which implies, FE = clrFE. Consequently, FE is a soft regular closed set in X. □ Theorem 3.16. Let FE be any soft gr-closed set in X. If FE e GE e clrFE then GE is also soft gr-closed set in X.
Proof. Let FE be a soft gr-closed set such that FE e GE e clrFE. So, clrGE e clrFE. We need to show that GE is also a soft gr-closed set. Let GE e HE, where HE is some open set in X, which implies clrFE e HE, since FE is a soft gr-closed set. Thus, clrGE e HE. Hence, GE is soft gr-closed. □
Theorem 3.17. The intersection of a soft gr-closed set and a soft closed set is always a soft generalized closed set.
Proof. Let FE be a soft gr-closed set and suppose GE be any soft open set in X, such that FE \ HE e GE, where HE is a soft closed set. We need to show that cl(FE \ HE) e GE. Now, FE \ HE e GE, implies FE e GE [HEc, or we can write clrFE e GE [HEc. Now, since we know that clFE e clrFE, then we have cl(FE \ HE) e clFE \ clHE e clrFE \ clHE = clrFE \ HE e GE. Hence, the proof is established. □
Remark 3.18. Let FE be a soft gr-closed set and GE be a soft regular closed set in (X; ; E), then FE \ GE is a soft gr-closed set in X.
From the above study we have the following diagram of one sided relations, whereas the reverse im-plications are not necessarily true.

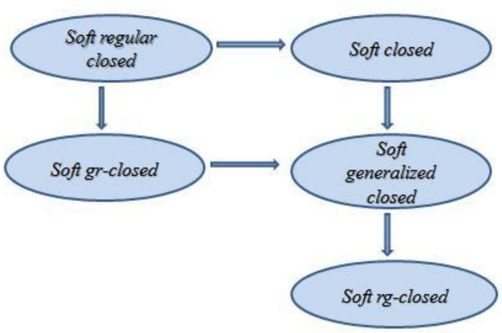

Fig 1.

Definition 3.19. A soft set FE of a soft space (X; ; E) is called soft gr-open if its complement is soft gr-closed.
Theorem 3.20. A soft set FE of X is soft gr-open iff GE e intrFE, whenever GE e FE and GE is soft closed.
Proof. Let FE be a soft gr-open set and GE be a soft closed set such that GE e FE, or we can write FEc e GcE, here GcE is open. Now, by our assumption FE is soft gr-open, then FEc is soft gr-closed. That is FEc = clrFE, which implies (intrFE)c = clrFEc e GcE. Hence, GE e intrFE. Conversely, Suppose FE is a soft set such that GE e intrFE, whenever GE is soft closed and FE e GE. Then we show that FE is a soft gr-open set. So, first we prove that FEc is a soft gr-closed set. Let FEc e HE, where HE is a soft open set in X. Also we can write, HEc e FE e intrFE (by assumption), which implies (intrFE)c e HE, or clrFEc e HE, which shows that FEc is a soft gr-closed set. Hence, FE is soft gr-open. □
Theorem 3.21. Let FE be any soft gr-open set in X. If intrFE e HE e FE then HE is also soft gr-open set in X.
Proof. Let FE be a soft gr-open set in X such that intrFE e HE e FE. Then FEc is soft gr-closed. So we can write, FEc e HEc e (intrFE)c = clrFEc. By Theorem 3.16, HEc is soft gr-closed set and hence, HE is soft gr-open set. □
Now, in the subsequent study, we remember the notion of Alexandroff space, which says that a topological space is an Alexandroff space if it is closed under arbitrary intersection. Hence, we define this in soft space as follows :
Definition 3.22. A soft topological space (X; ; E) is said to be soft Alexandroff space if it is closed under arbitrary intersection.
Definition 3.23. Let (X; ; E) be a soft topological space. Let FE be a soft subset of the space X.
Then fFE g is defined by fFE g = fGE j FE e GE; where GE 2 O(X; ; E)g. If the space X is soft Alexandroff space then (FE) is soft open in X.
Theorem 3.24. Let FE be a soft set in a soft Alexandroff space (X; ; E), then FE is soft gr-closed set in X if clr (FE).
Proof. Let FE be a soft gr-closed set in X, then there exists a soft open set GE of X containing FE such that clr(FE) e GE (since, the space is soft Alexandroff space, so, FE e (FE), an open set in X). Consequently, clr(FE) = (FE). □
Theorem 3.25. Let FE be a soft -set in a soft Alexandroff space (X; ; E). Then FE is a soft gr-closed subset of X if and only if it is a soft regular closed set in X.
Proof. Let FE be a soft gr-closed subset of X, then clr(FE) e (FE). But FE is soft -set, so we can write, clr e (FE) = FE. Also, FE e clr(FE), this means FE is soft regular closed in X. □

4. SOFT GENERALIZED REGULAR CONTINUOUS FUNCTIONS

In this section, we define some soft generalized continuous functions in soft topological spaces and establish some properties related to these functions and also the interrelationships among.
Definition 4.1. Let (X; ; E1) and (Y; ; E2) be two soft topological spaces. A soft function f : X ! Y is said to be

1) a soft generalized regular continuous function if the inverse of every soft closed set in Y is soft gr-closed in X.
2) a soft generalized continuous function if inverse of every soft closed set in Y is soft generalized closed in X.
3) a soft regular generalized continuous function if the inverse of every soft closed set in Y is soft regular generalized closed (briefly, rg-closed) in X.
4) a soft regular continuous function if the inverse of every soft closed set in Y is soft regular closed in X.
5) a soft generalized irresolute if the inverse of every soft generalized closed set in Y is soft generalized closed in X.
6) a soft regular generalized irresolute if the inverse of every soft rg-closed set in Y is soft rg-closed in X.
7) a soft generalized regular irresolute if inverse image of every soft gr-closed set in Y is soft gr-closed set in X.

Remark 4.2. Soft gr-continuous function and soft continuous function are independent of each other.
This is demonstrated by the following example.
Example 4.3. (1). Let X = Y = fa; b; cg with the parameter sets E1 = E2 = fe1; e2g and = fe; Xeg,
= fe; Ye ; G1E2 g, where G1E2 is a soft set on Y defined by G1(e1) = fag G1(e2) = f g
Here, (X; ; E1) and (Y; ; E2) is a soft topological space. Let f : X ! Y be a soft function defined by
f(a) = a; f(b) = b; f(c) = c: Here, we can see that f is not soft continuous but is soft gr-continuous.
(2). Let us take two new topologies defined over X and Y respectively as, 0 = fX;e e; FE11 ; FE21 ; FE31 g and
00 = fYe ; e; GE2 g, where FE11 ; FE21 ; FE31 ; GE1 are soft sets in X and Y defined by F 1(e1) = fa; bg; F 1(e2) = f g
F 2(e1) = fbg; F 2(e2) = fXg F 3(e1) = fb; cg; F 3(e2) = f g and G(e1) = fb; cg; G(e2) = f g.
Defining f : X ! Y by f(a) = a; f(b) = b; f(c) = c: Here, it can be easily seen that f is soft continuous but is not a soft gr-continuous.
Theorem 4.4. Every soft gr-continuous function is soft generalized continuous.
Proof. Obvious. □
Remark 4.5. A soft generalized continuous function is not a soft gr-continuous function, we can see from Example 4.3(2) that f is not soft gr-continuous but is soft generalized continuous.
Theorem 4.6. Every soft regular continuous function is soft gr-continuous.
Proof. Let FE be a soft closed set in Y . Since, f is soft regular continuous, so f 1(FE) is soft regular closed in X. Also, every soft regular closed set is soft gr-closed set, so f 1(FE) is soft gr-closed in X. Hence, f is soft gr-continuous. □
Remark 4.7. A soft gr-continuous function is not a soft regular continuous function, which can be clearly verified by Example 4.3(1).

Theorem 4.8. Every soft continuous function is a soft rg-continuous function.
Proof. Let FE be a soft closed set in Y . Since, f is soft continuous, so f 1(FE) is soft closed in X. But every soft closed set is soft rg-closed, thus f 1(FE) is soft rg-closed in X. Hence, f is soft rg-continuous. □
Remark 4.9. A soft rg-continuous function is not always a soft continuous function. This can be observed by Example 4.3(1)
Theorem 4.10. Every soft gr-continuous function is a soft rg-continuous function.
Proof. Let FE be a soft closed set in Y . Since, f is soft gr-continuous, so f 1(FE) is soft gr-closed set in X. But every soft gr-closed set is soft rg-closed set. Thus, f 1(FE) is soft rg-closed set. Hence, f is soft rg-continuous. □
Remark 4.11. A soft rg-continuous function is not a soft gr-continuous function, which can be verified by Example 4.3(2).
Now, we depict the relation studied above in the following diagram:

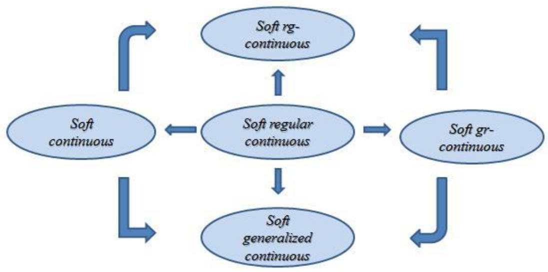

Fig 2.

We conclude this section by the compositions of all these functions in the last theorem.
Theorem 4.12. Let (X; ; E1); (Y; ; E2) and (Z; ; E3) be soft topological spaces. Let f : X ! Y and
g : Y ! Z be two soft functions, then if

1) if f is soft gr-continuous and g is soft continuous then g f is soft gr-continuous.
2) if f is soft gr-continuous and g is soft regular continuous then g f is soft gr-continuous.
3) if f is soft gr-irresolute and g is soft gr-continuous then g f is soft gr-continuous.
4) if f and g are soft gr-irresolute then g f is soft gr-irresolute.

Proof. (1) Let GE be a soft closed set in Z. Then g 1(GE) is soft closed in Y , as g is soft continuous. Also, f is soft gr-continuous, implies f 1(g 1(GE)) is soft gr-closed in X. Thus, (g f) 1. Consequently, g f is a soft gr-continuous function.
(2) Let GE be a soft closed set in Z. Since, g is soft regular continuous then g 1(GE) is soft regular closed in Y . As every soft regular closed set is soft closed, implies g 1(GE) is soft closed in Y . Also, f is soft gr-continuous, this shows f 1(g 1(GE)) is soft gr-closed set in X. Thus, (f g) 1(GE) is soft gr-closed in X. Hence, g f is soft gr-continuous function.
(3) Let GE be a soft closed set in Z. Sine, g is soft gr-continuous, so, g 1(GE) is soft gr-closed set in Y . Also, f is soft gr-irresolute, implies f 1(g 1(GE)) is soft gr-closed set in X. Thus, (g f) 1(GE) is soft gr-closed in X. Hence, g f is soft gr-continuous.
(4) Let GE be a soft gr-closed set in Z. Since, g is soft gr-irresolute, so g 1(GE) is soft gr-closed in Y .
Also, f is soft gr-irresolute, implies f 1(g 1(GE)) is soft gr-closed in X. Thus, (g f) 1(GE) is soft gr-closed in X. Hence, g f is soft gr-irresolute.

□

5. CONCLUSION

In this paper, we initiated the notion of soft generalized regular closed set in a soft topological space and established some relations between the soft gr-closed sets and several other kinds of soft generalized sets such as soft regular closed sets, soft generalized closed sets and also with that of soft rg-closed sets. Thereafter, some implications were drawn and some reverse implications which are not applicable are also illustrated with the help of counter examples. Moreover, some applications of these sets were also drawn via some soft generalized continuities. However, the notion of soft gr-closed sets can be collaborated with the concept of fuzzy soft topological spaces.

6. REFERENCES

[1] Balachandran, K. (1991). On generalized continuous maps in topological spaces. Mem. Fac. Sci. Kochi Univ. Ser. A Math., 12, 5-13.
[2] Bhattacharya, B., & Chakraborty, J. (2015). Generalized regular fuzzy closed sets and their Appli-cations. J. Fuzzy Math, 23(1), 227-239.
[3] Bhattacharya, S. (2011). On generalized regular closed sets. Int. J. Contemp. Math. Sciences, 6(3), 145-152.
[4] Hussain, S., & Ahmad, B. (2011). Some properties of soft topological spaces. Computers & Mathe-matics with Applications, 62(11), 4058-4067.
[5] Kannan, K. (2012). Soft generalized closed sets in soft topological spaces. Journal of theoretical and applied information technology, 37(1), 17-21.
[6] Levine, N. (1970). Generalized closed sets in topology. Rendiconti del Circolo Matematico di Palermo, 19(1), 89-96.
[7] Maji, P. K., Biswas, R., & Roy, A. (2003). Soft set theory. Computers & Mathematics with Applica-tions, 45(4-5), 555-562.
[8] Min, W. K. (2011). A note on soft topological spaces. Computers & Mathematics with Applications, 62(9), 3524-3528.
[9] Molodtsov, D. (1999). Soft set theory - first results. Computers & Mathematics with Applications, 37(4-5), 19-31.
[10] N. Palaniappan and K. C. Rao; Regular generalized closed sets; Kyungpook Math 33(2)(1993); 211 – 219
[11] Shabir, M., & Naz, M. (2011). On soft topological spaces. Computers & Mathematics with Applica-tions, 61(7), 1786-1799.
[12] Yuksel, S. A., Tozlu, N., & Ergul, Z. G. (2014). Soft regular generalized closed sets in soft topological spaces. Int. Journal of Math. Analysis, 8(8), 355-367.